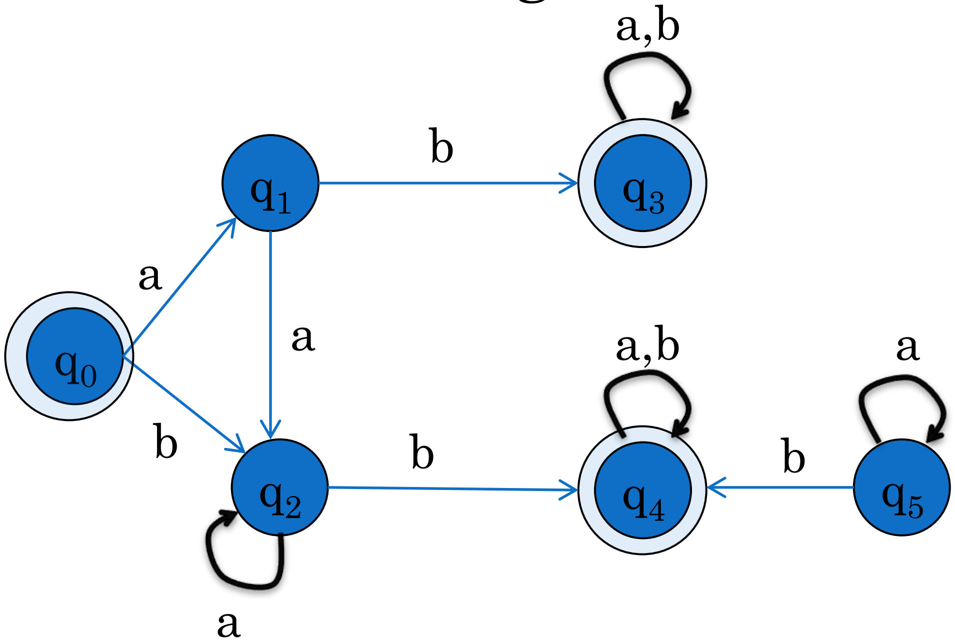
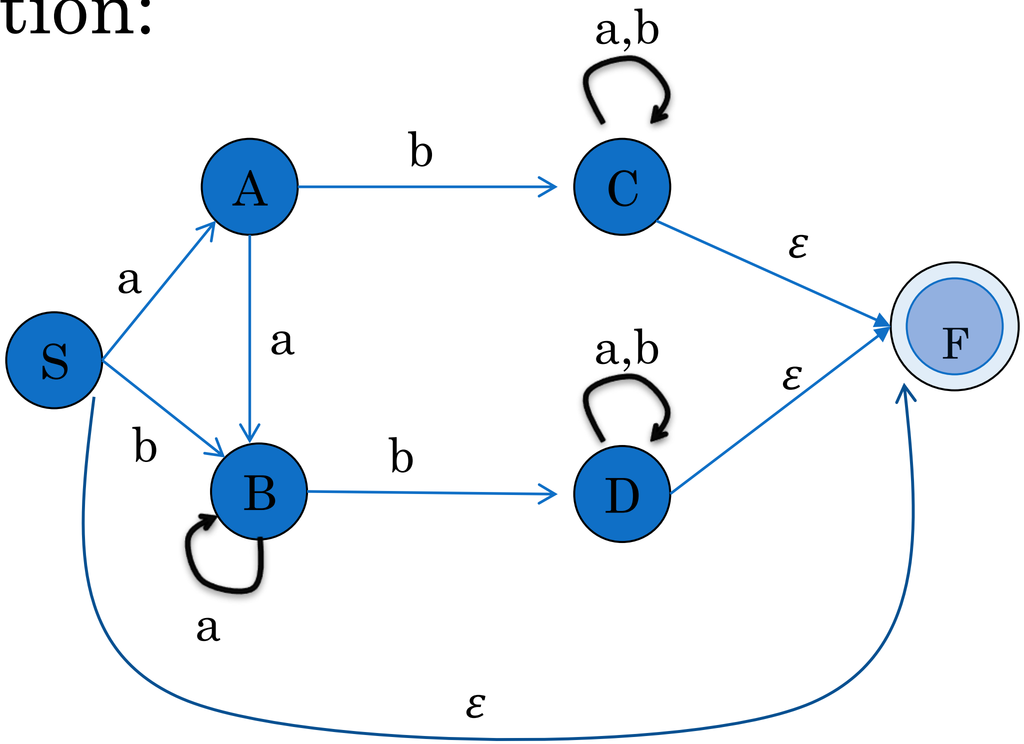
Quiz

- For the below FA, write left linear and write linear grammar.



- Solution:



$$S \to F|aA|bB$$
$$A \to aB|bC$$
$$B \to aB|bD$$
$$C \to aC|bC|F$$
$$D \to aD|bD|F$$
$$F \to \varepsilon$$

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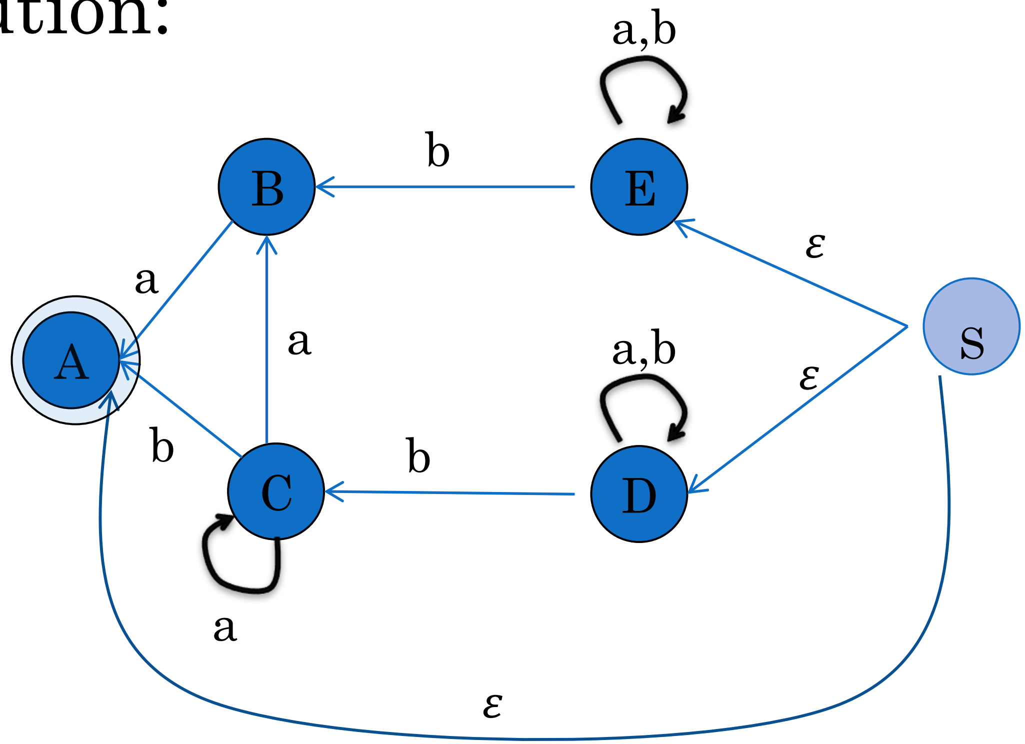

$$S \rightarrow A|E|D$$
$$A \rightarrow \varepsilon$$
$$B \rightarrow Aa$$
$$C \rightarrow Ca|Ab|Ba$$
$$D \rightarrow Da|Db|Cb$$
$$E \rightarrow Ea|Eb|Bb$$

Formal languages and automata

Identifying Nonregular Languages

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Regular or not?

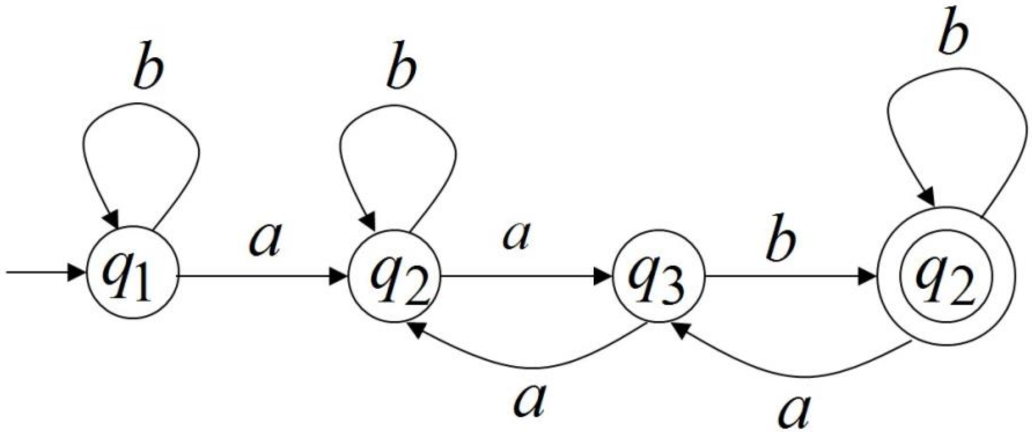
- Given language L how can we check if it is not a regular language ?
  - The answer is not obvious.
  - Not being able to design a DFA does not constitute a proof!

The Pigeonhole principle

- If there are n pigeons and m holes and $n > m$, then at least one hole has > 1 pigeons.

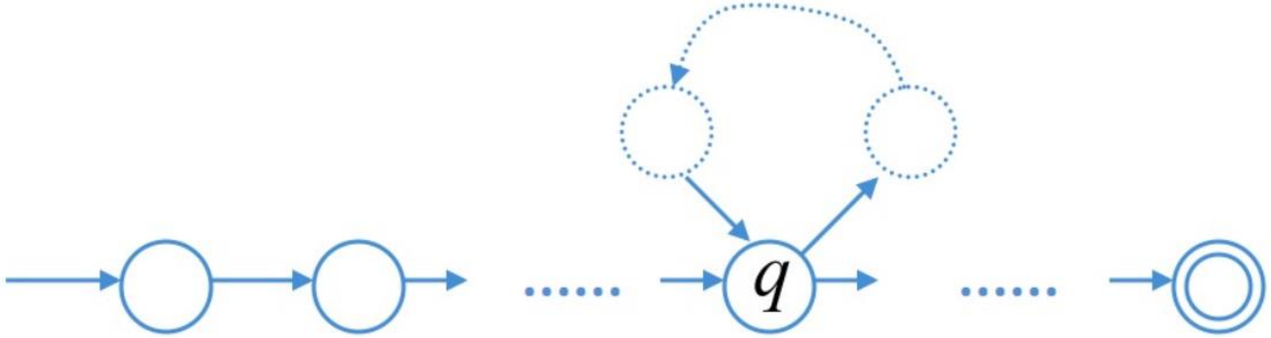
- What do pigeons have to do with regular languages?

- Consider the DFA



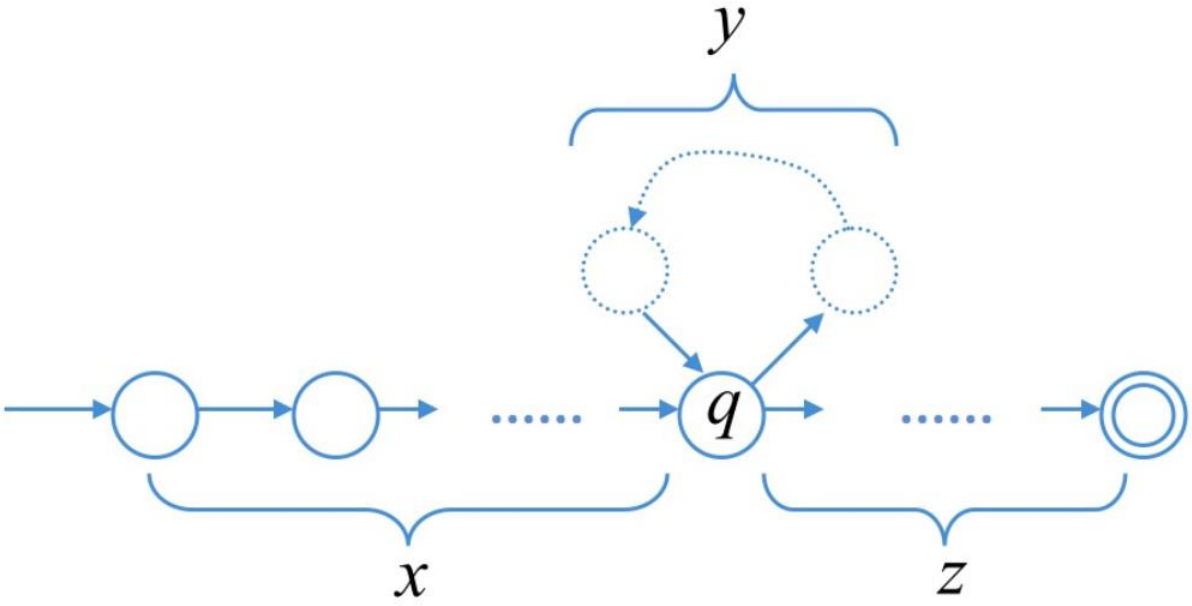
- With strings *a*, *aa* or *aab*, no state is repeated
- With strings *aabb*, *bbaa*, *abbabb* or *abbbabbabb*, a state is repeated
- In fact, for any ω where $|\omega| \geq 4$, some state has to repeat? Why?

- When traversing the DFA with the string ω, if the number of transitions $\geq$ number of states, some state q has to repeat!
- Transitions are pigeons, states are holes.



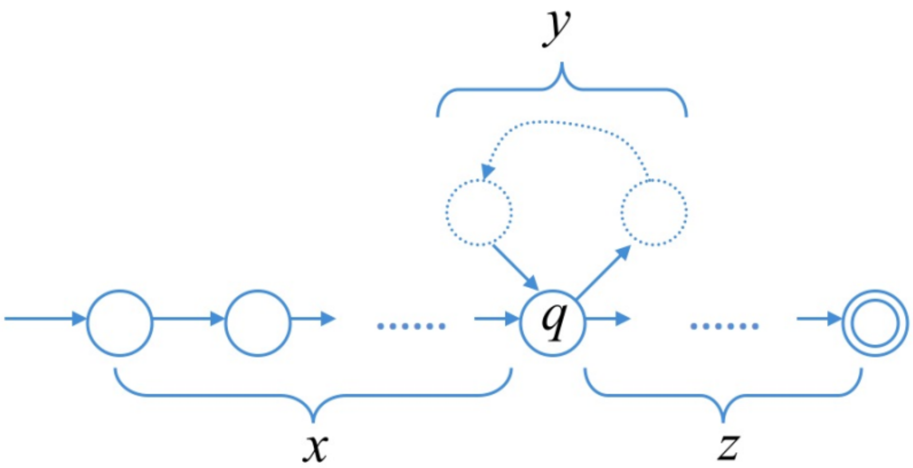
Pumping a string

- Consider a string $\omega = xyz$



- $|y| \geq 1$
- $|xy| \leq m$ (m the number of states)

- Consider a string $\omega = xyz$



- If $\omega = xyz \in L$ that so are xy^iz for all $i \geq 0$
- The substring y can be pumped.
- So if a DFA accepts a sufficiently long string, then it accepts an infinite number of strings!

The Pumping Lemma

LEMMA

Given an infinite regular language L

1. *There exists an integer m such that*
2. *for any string* $\omega \in L$ *with length* $|\omega| \geq m$,
3. *we can write* $\omega = xyz$ *with* $|y| \geq 1$ *and* $|xy| \leq m$,
4. *such that the strings* xy^iz *for* $i = 0, 1, 2 \ldots$ *are also in L*

Thus any sufficiently long string can be "pumped."

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- Assume L is regular
- We must pick an string s$\in L$ which |s|>m
- Let s = $a^m b^m$, |$a^n b^n$|=2m>m
- Let s=XYZ
- Let a^m=XY, b^m=Z. So, Y is a string of ‘a’s.
- It is confirmed that|XY|<=m and |Y|>=1

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  - It definitely belongs to L
  - Its length is odd
- XYZ=$a^m b a^m$. XY=a^m which |XY|<=m.
- Therefore, Y can be 'a^k' where 1<=k<=m
- XYYZ=$a^{m+k} b a^m$ ※

- In other word, When we have to remember and count with infinity, So the corresponding language can not be regular!
- Consider $L = \{a^n b^m\}$

$$n = m$$

$$n \neq m$$

$$n \leq m$$

$$n, m > 0$$

Consider $L = \{\omega | n_a(\omega) < n_b(\omega)\}$

Consider $L = \{1^{n^2} | n \geq 0\}$

- Consider $L = \{a^n b^m c^n \mid n, m > 0\}$

- Consider $L = \{a^i b^j c^k \mid \ldots.\}$

$$i, j, k > 0$$

$$i + j = k$$

- Consider $L = \{a^{2n}b^{2n} \mid n < 1000\}$

- Consider $L = \left\{a^i b^j c^k \,\middle|\, i + j < k \ and \ k < 10\right\}$

- Consider $L = \{a^i b^j c^k \mid i + j < k \ and \ k = 10\}$

- Consider $L = \{a^i b^j c^k \mid i + j > k \; and \; k = 10\}$

- Consider $L = \left\{a^i b^j c^k \,\middle|\, i + j < k \text{ } and \text{ } k > 10\right\}$

- Consider $L = \{a^n b^m \mid n + m = 2k + 1\}$

- Consider $L = \{WW^R \mid W \in \Sigma^*\}$

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- Consider $L = \{WW \mid W \in \Sigma^*\}$

- Consider $L = \{UWW^RV \mid U, V, W \in \Sigma^*\}$

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- It is regular!, because for each input string, we can suppose w = λ
- Let aababaaab
- Let aabab aaab

- Consider $L = \{UWW^RV \mid U, V, W \in \Sigma^+\}$

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- It is regular and we just find ‘aa’ or ‘bb’ in the input string.

- Consider $L = \{WW^RV \mid V, W \in \Sigma^+\}$

- $L = \{a^i b^j c^k | i - j < k, k < 10\}$

- $L = \{a^n b^i | n - i = 2\}$

- $L = \left\{a^n b^i \middle| |n - i| = 2\right\}$
- $L = \{a^n | n > 1000\ or\ n\ is\ prime\}$

True/False

True/False

If L_1 is not regular and L_2 is regular then $L = L_1 L_2 = \{xy : x \in L_1 \text{and } y \in L_2\}$ is not regular.

$L = \{a^i b^j a^k : i + k < 10 \text{ and } j > 10\}$ is not regular.

$L = \{a^i b^j : i + j \geq 10\}$ is not regular.

$L = \{w \in \{a, b\}^* : n_a(w) \times n_b(w) = 0 \bmod 2\}$ is regular.

If $L_1 \cap L_2$ is regular then L_1 and L_2 are regular.

If $L_1 \subseteq L_2$ and L_2 is regular, then L_1 must be regular.

If L_1 and L_2 are nonregular, then $L_1 \cup L_2$ must be nonregular.

There are subsets of a regular language which are not regular.

If L is nonregular then $\overline{L}$ is nonregular.