

# Formal languages and automata

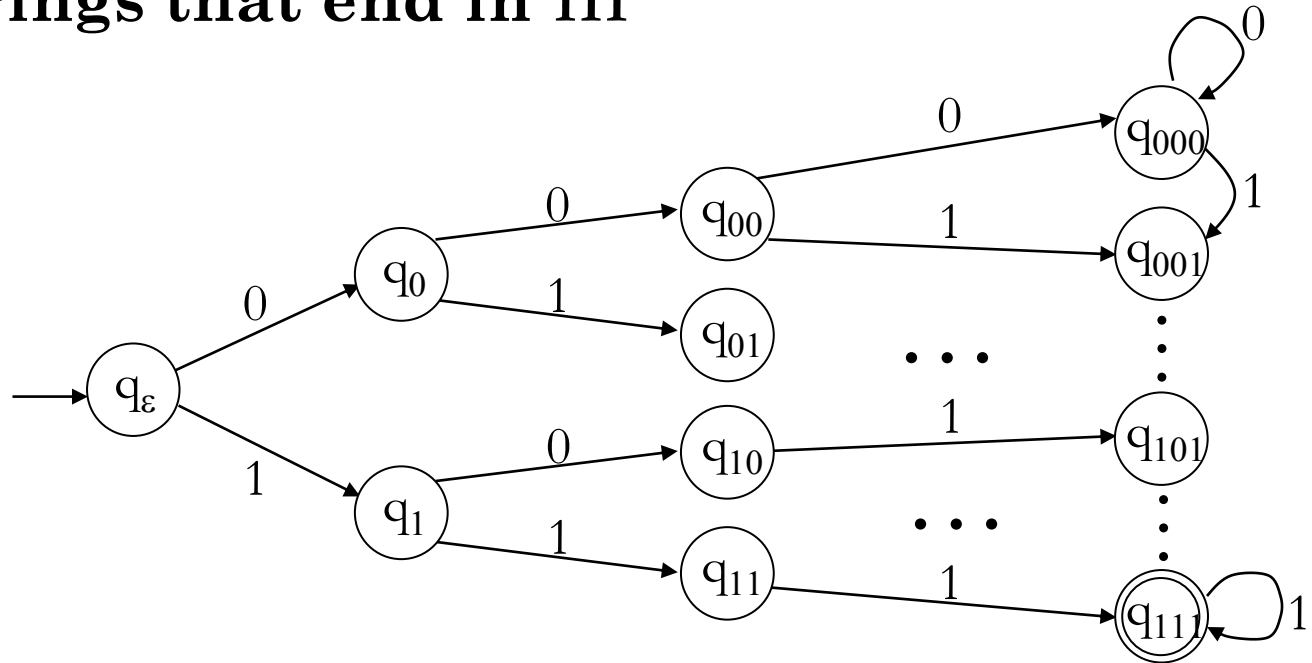
DFA minimization algorithm

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# Example

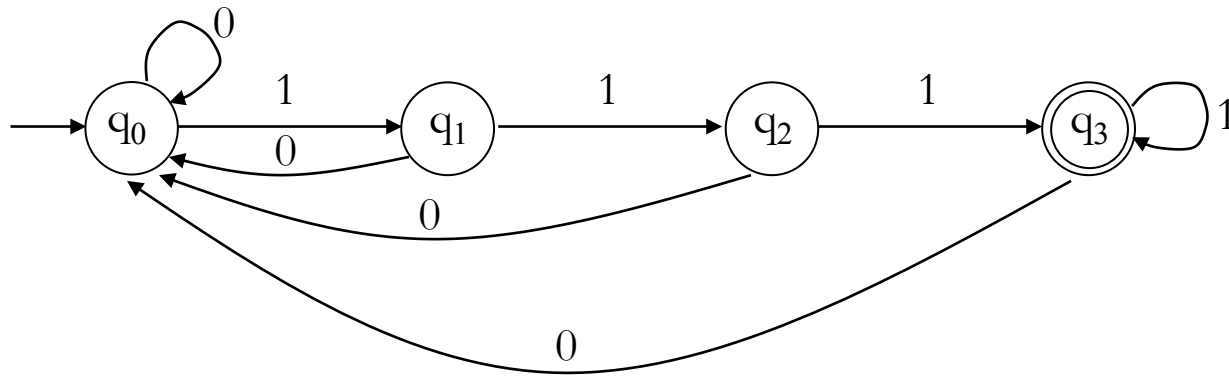
- Construct a DFA over alphabet  $\{0, 1\}$  that accepts those strings that end in 111



- This is big, isn't there a **smaller** DFA for this?

# Smaller DFA

- Yes, we can do it with 4 states:

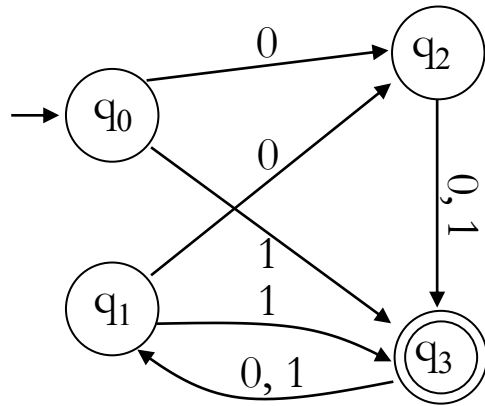


- The state remembers the number of consecutive 1s at the end of the string (up to 3)

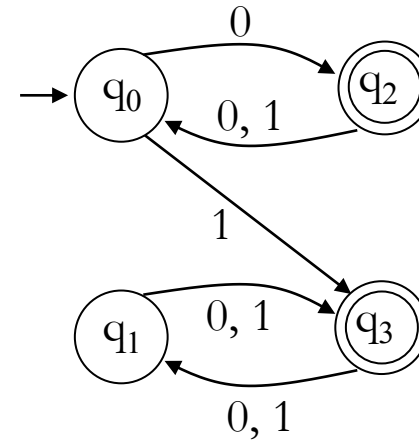
# DFA minimization

- There is an **algorithm** to start with any DFA and reduce it to the smallest possible DFA
- The algorithm attempts to identify classes of **equivalent states**
- These are states that can be **merged together** without affecting the answer of the computation

# Examples of equivalent states



$q_0, q_1$  equivalent



$q_0, q_1$  equivalent

$q_2, q_3$  also equivalent

# Equivalent and distinguishable states

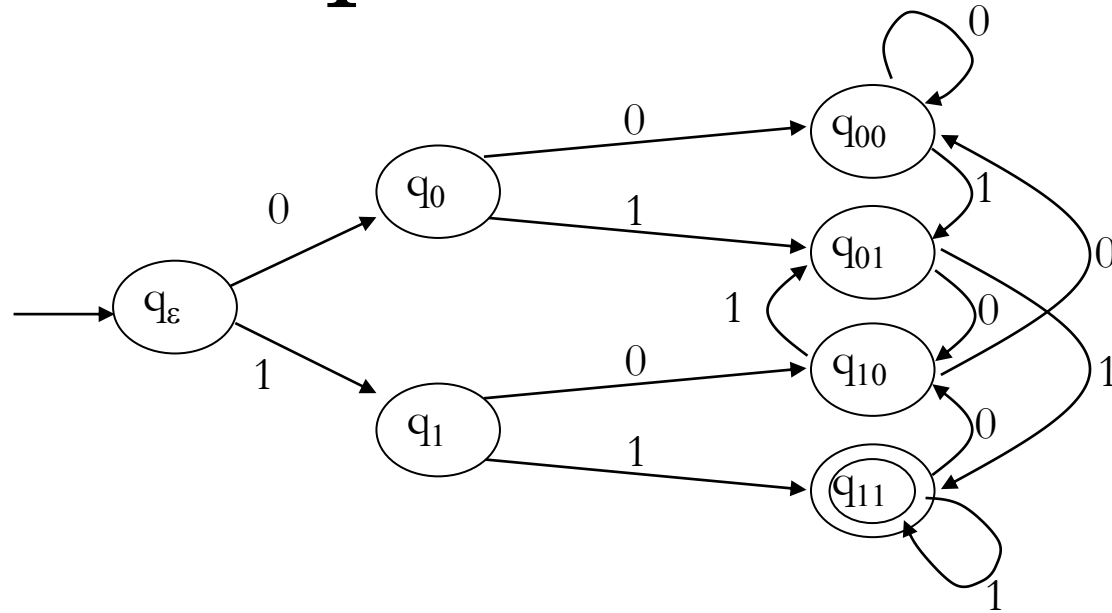
- Two states  $q, q'$  are **equivalent** if

For every string  $w$ , the states  $\delta(q, w)$  and  $\delta(q', w)$  are either both accepting or both rejecting

- Here,  $\delta(q, w)$  is the state that the machine is in if it starts at  $q$  and reads the string  $w$
- $q, q'$  are distinguishable if they are not equivalent:

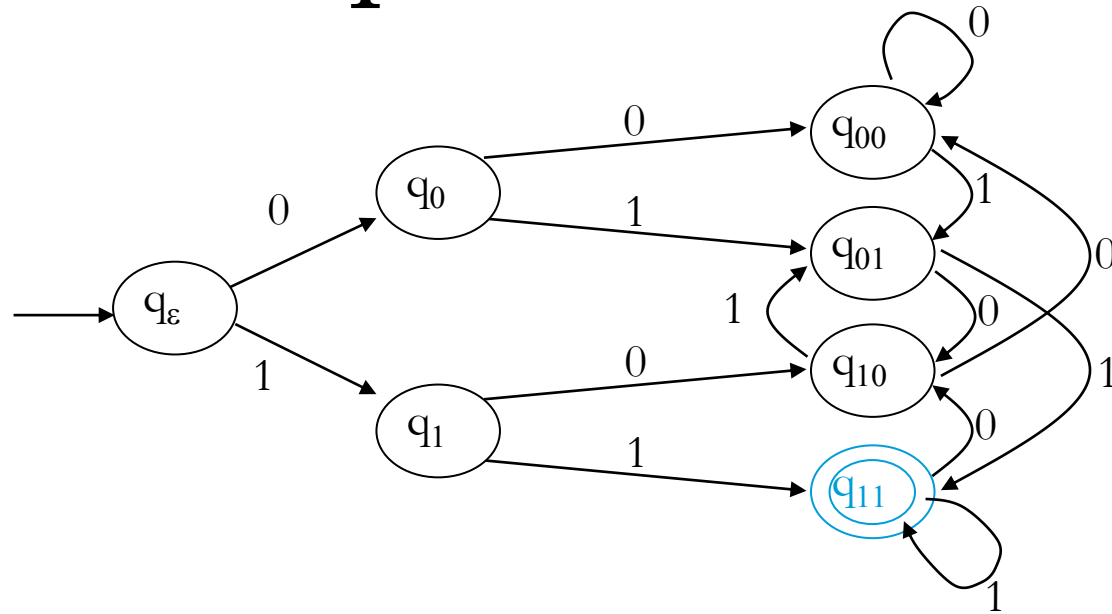
For some string  $w$ , one of the states  $\delta(q, w), \delta(q', w)$  is accepting and the other is rejecting

# Example of DFA minimization



|           |              |       |       |           |           |           |
|-----------|--------------|-------|-------|-----------|-----------|-----------|
| $q_0$     |              |       |       |           |           |           |
| $q_1$     |              |       |       |           |           |           |
| $q_{100}$ |              |       |       |           |           |           |
| $q_{101}$ |              |       |       |           |           |           |
| $q_{110}$ |              |       |       |           |           |           |
| $q_{111}$ |              |       |       |           |           |           |
|           | $q_\epsilon$ | $q_0$ | $q_1$ | $q_{100}$ | $q_{101}$ | $q_{110}$ |

# Example of DFA minimization

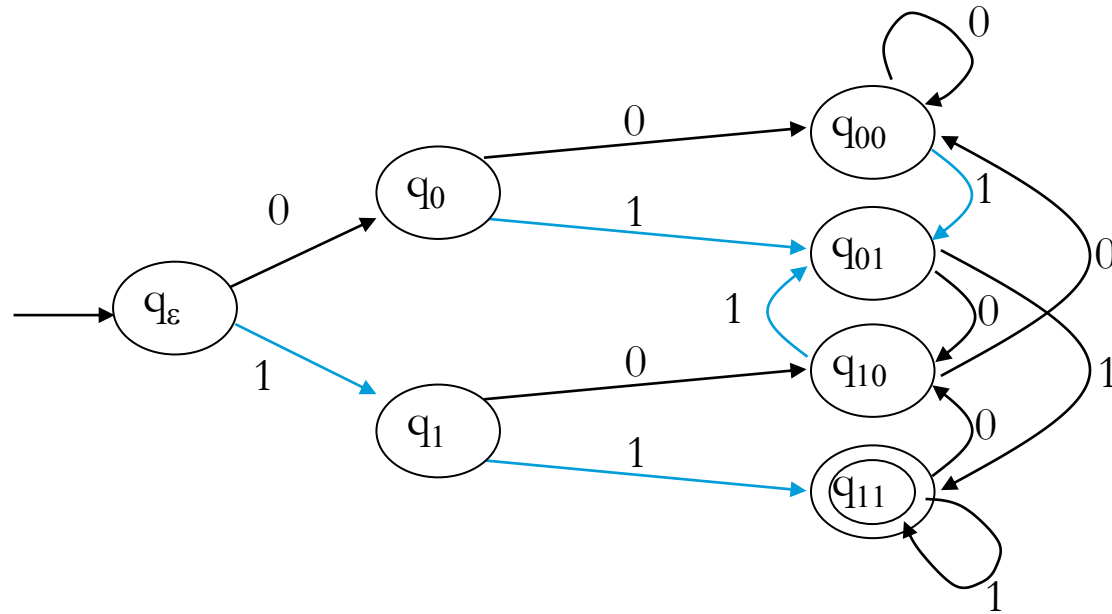


|           |              |       |       |           |           |           |
|-----------|--------------|-------|-------|-----------|-----------|-----------|
| $q_0$     |              |       |       |           |           |           |
| $q_1$     |              |       |       |           |           |           |
| $q_{100}$ |              |       |       |           |           |           |
| $q_{101}$ |              |       |       |           |           |           |
| $q_{110}$ |              |       |       |           |           |           |
| $q_{111}$ | x            | x     | x     | x         | x         | x         |
|           | $q_\epsilon$ | $q_0$ | $q_1$ | $q_{100}$ | $q_{101}$ | $q_{110}$ |

①  $q_{111}$  is distinguishable from all other states



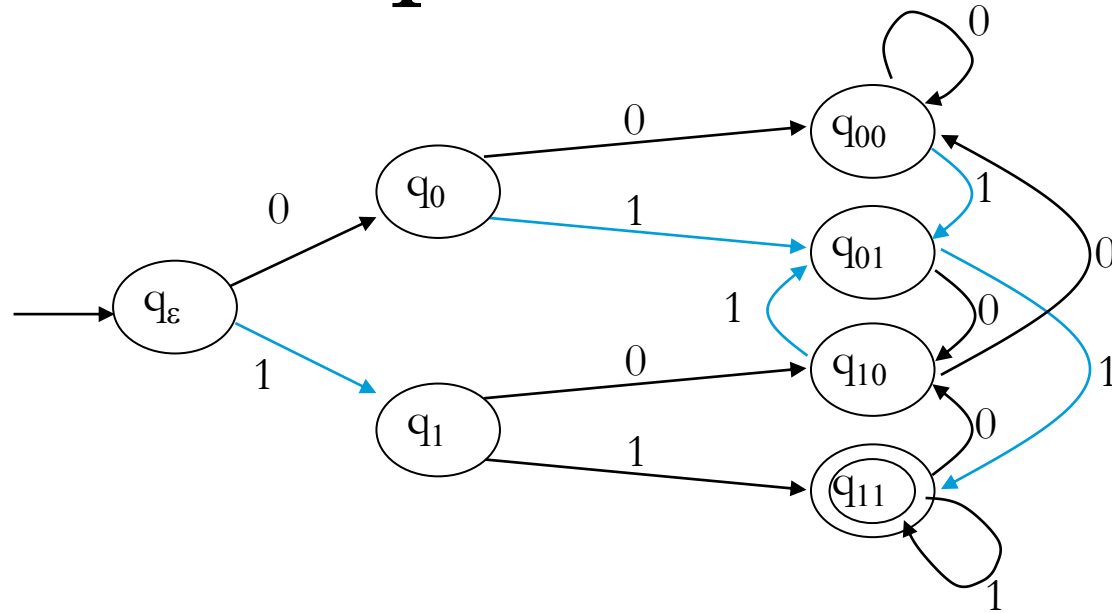
# Example of DFA minimization



|          |              |       |       |          |          |          |
|----------|--------------|-------|-------|----------|----------|----------|
| $q_0$    |              |       |       |          |          |          |
| $q_1$    | x            | x     |       |          |          |          |
| $q_{00}$ |              |       | x     |          |          |          |
| $q_{01}$ |              |       |       |          |          |          |
| $q_{10}$ |              |       | x     |          |          |          |
| $q_{11}$ | x            | x     | x     | x        | x        |          |
|          | $q_\epsilon$ | $q_0$ | $q_1$ | $q_{00}$ | $q_{01}$ | $q_{10}$ |

- ②  $q_1$  is distinguishable from  $q_\epsilon$ ,  $q_0$ ,  $q_{00}$ ,  $q_{10}$   
 On transition 1, they go to distinguishable states

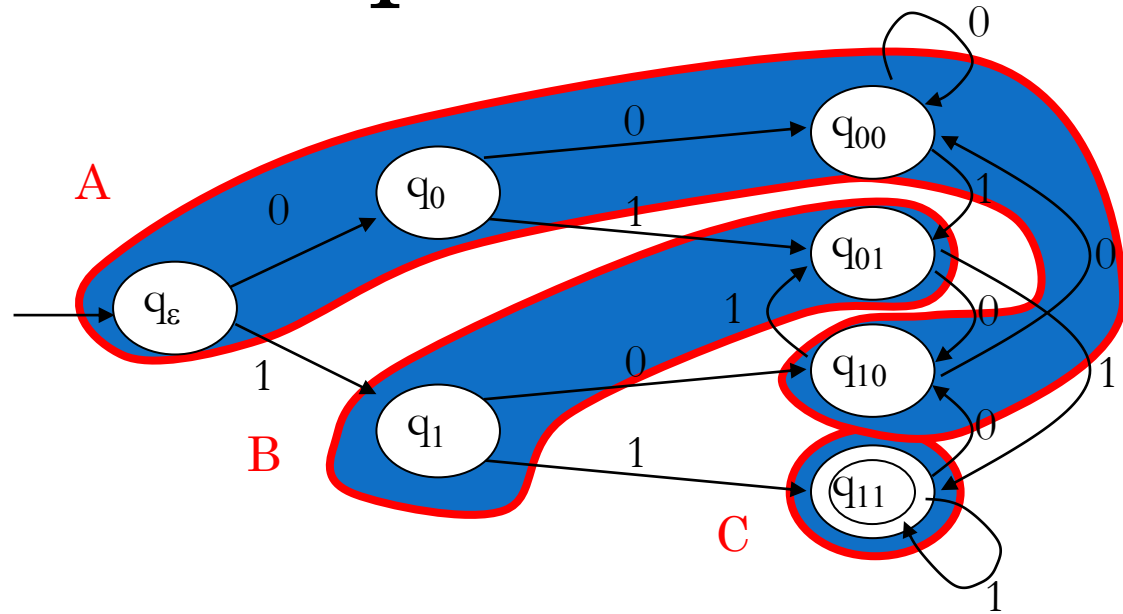
# Example of DFA minimization



|          |              |       |       |          |          |          |
|----------|--------------|-------|-------|----------|----------|----------|
| $q_0$    |              |       |       |          |          |          |
| $q_1$    | x            | x     |       |          |          |          |
| $q_{00}$ |              |       | x     |          |          |          |
| $q_{01}$ | x            | x     |       | x        |          |          |
| $q_{10}$ |              |       | x     |          | x        |          |
| $q_{11}$ | x            | x     | x     | x        | x        | x        |
|          | $q_\epsilon$ | $q_0$ | $q_1$ | $q_{00}$ | $q_{01}$ | $q_{10}$ |

- ②  $q_{01}$  is distinguishable from  $q_\epsilon$ ,  $q_0$ ,  $q_{00}$ ,  $q_{10}$   
 On transition 1, they go to distinguishable states

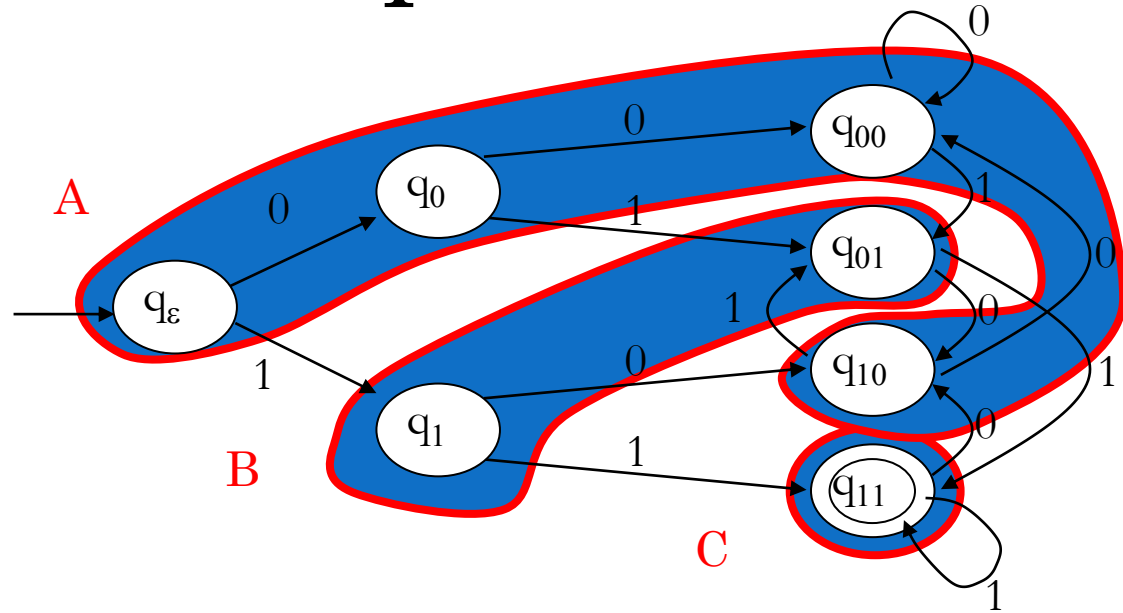
# Example of DFA minimization



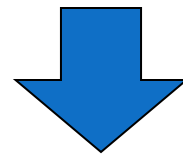
|          |              |       |       |          |          |          |
|----------|--------------|-------|-------|----------|----------|----------|
| $q_0$    | A            |       |       |          |          |          |
| $q_1$    | x            | x     |       |          |          |          |
| $q_{00}$ | A            | A     | x     |          |          |          |
| $q_{01}$ | x            | x     | B     | x        |          |          |
| $q_{10}$ | A            | A     | x     | A        | x        |          |
| $q_{11}$ | x            | x     | x     | x        | x        | x        |
|          | $q_\epsilon$ | $q_0$ | $q_1$ | $q_{00}$ | $q_{01}$ | $q_{10}$ |

- ③ Merge states not marked distinguishable
- $q_\epsilon, q_0, q_{00}, q_{10}$  are equivalent  $\rightarrow$  group A
  - $q_1, q_{01}$  are equivalent  $\rightarrow$  group B
  - $q_{11}$  cannot be merged  $\rightarrow$  group C

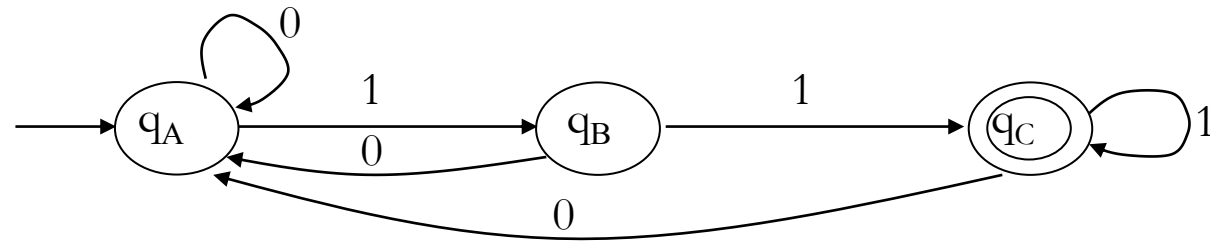
# Example of DFA minimization



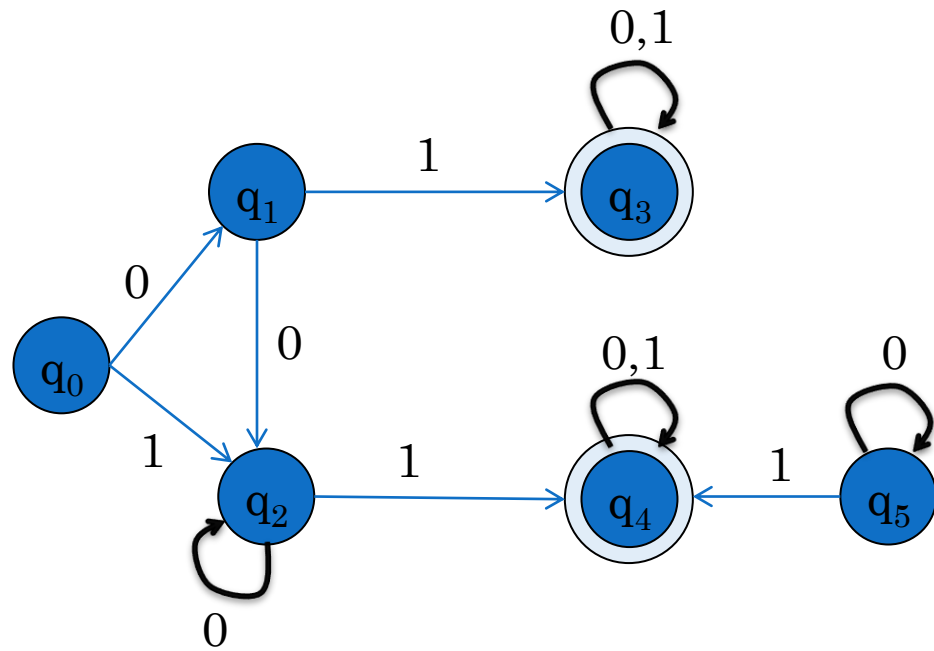
|           |              |       |       |           |           |           |
|-----------|--------------|-------|-------|-----------|-----------|-----------|
| $q_0$     | A            |       |       |           |           |           |
| $q_1$     | x            | x     |       |           |           |           |
| $q_{100}$ | A            | A     | x     |           |           |           |
| $q_{101}$ | x            | x     | B     | x         |           |           |
| $q_{110}$ | A            | A     | x     | A         | x         |           |
| $q_{111}$ | x            | x     | x     | x         | x         | x         |
|           | $q_\epsilon$ | $q_0$ | $q_1$ | $q_{100}$ | $q_{101}$ | $q_{110}$ |



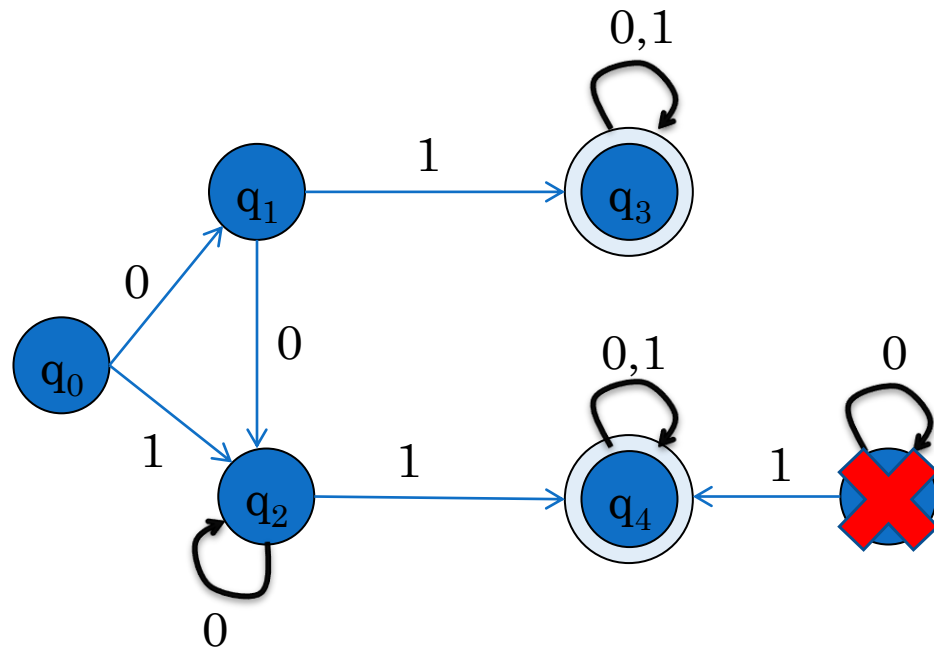
minimized DFA:



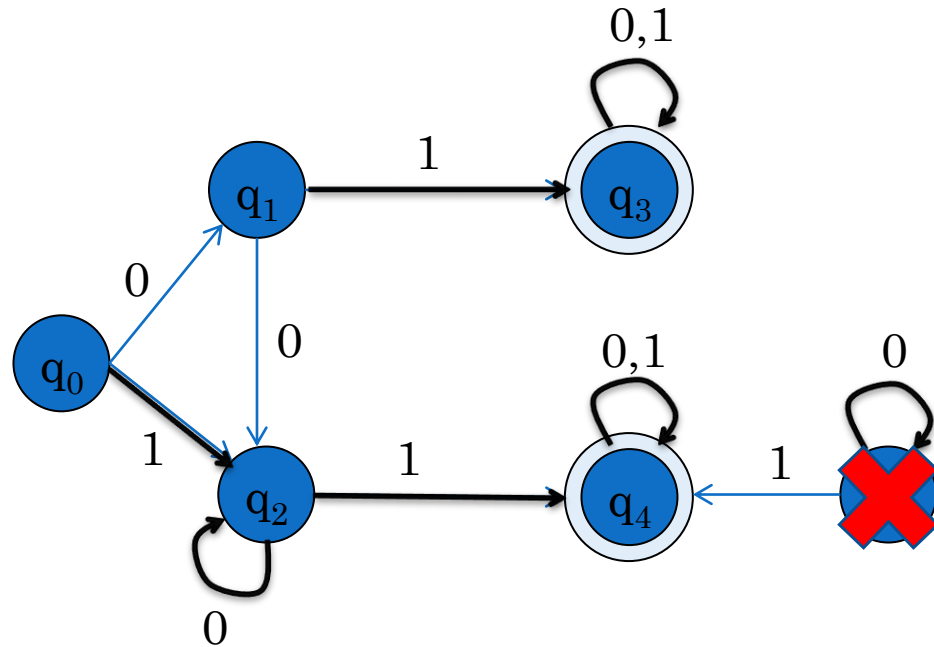
# Example2



# Example2



# Example2

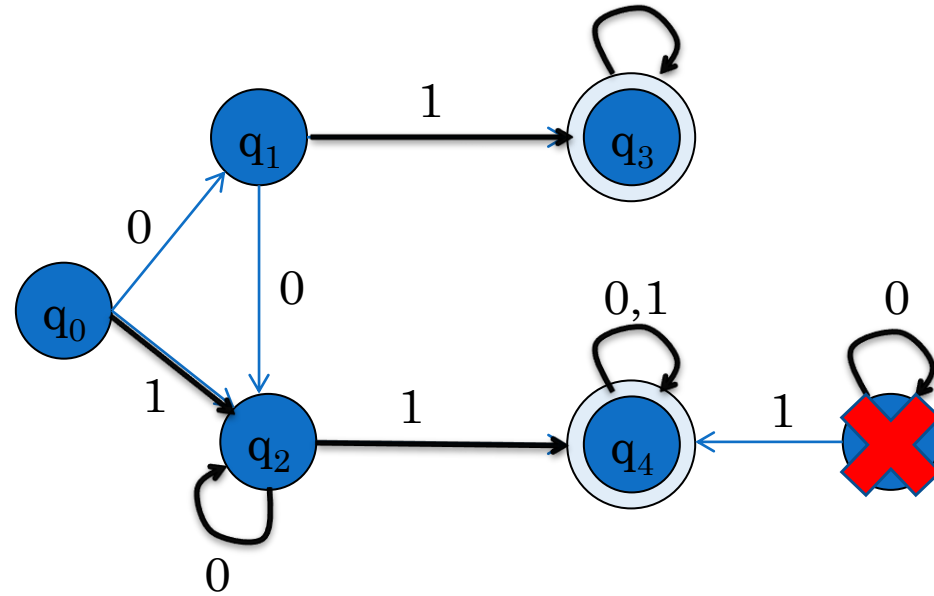
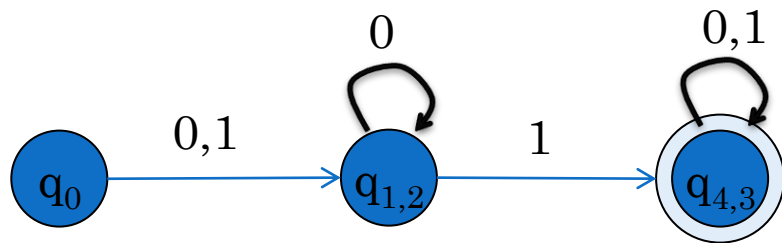


$G1 = \{q3, q4\}$

$G2 = \{q0, q1, q2\}$

$q0$  is distinguishable, So we have:  $G21 = \{q0\}$ ,  $G22 = \{q1, q2\}$

# Example2



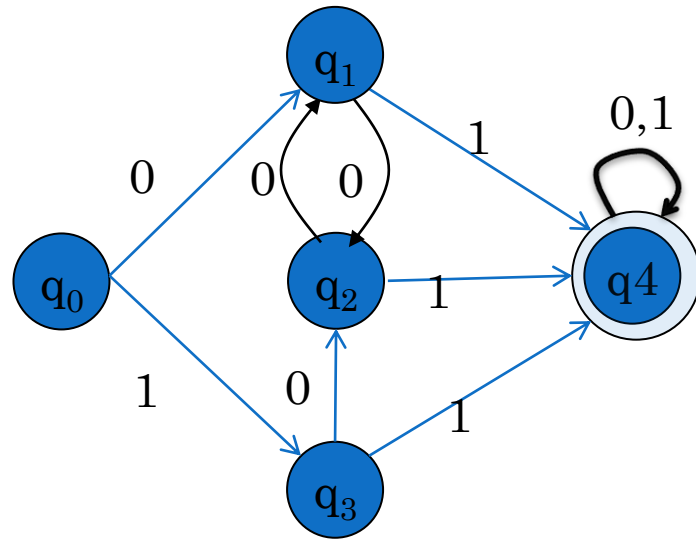
$G1 = \{q3, q4\}$

$G2 = \{q0, q1, q2\}$

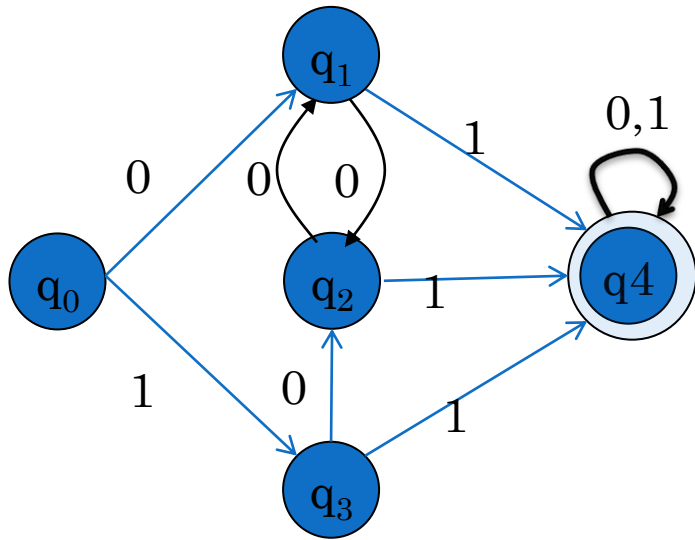
$q0$  is distinguishable, So we have:  $G21 = \{q0\}$ ,  $G22 = \{q1, q2\}$



# Example 3



# Example 3

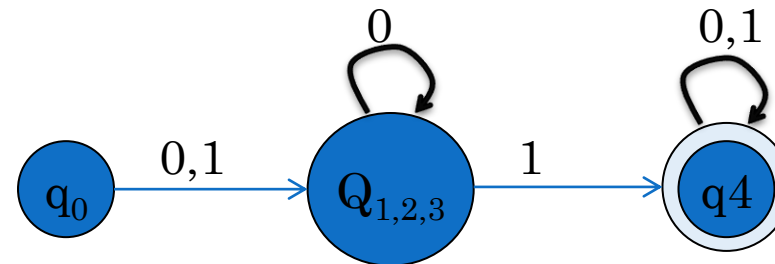


$G1 = \{q_0, q_1, q_2, q_3\}$

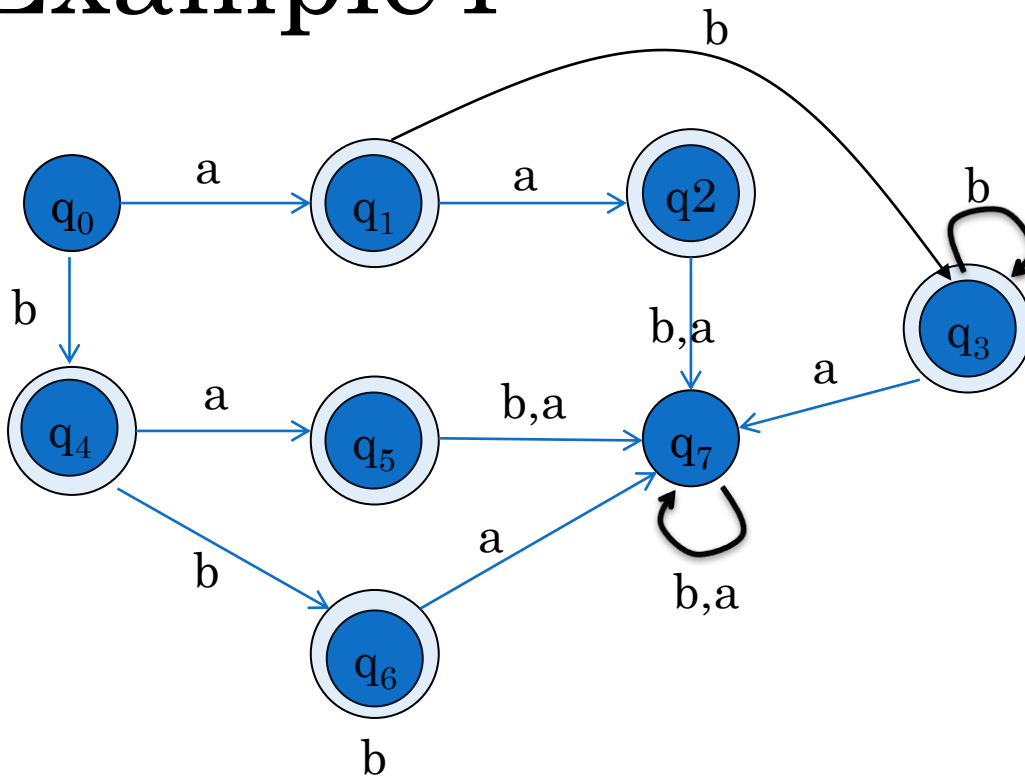
$G2 = \{q_4\}$

$q_0$  is distinguishable,

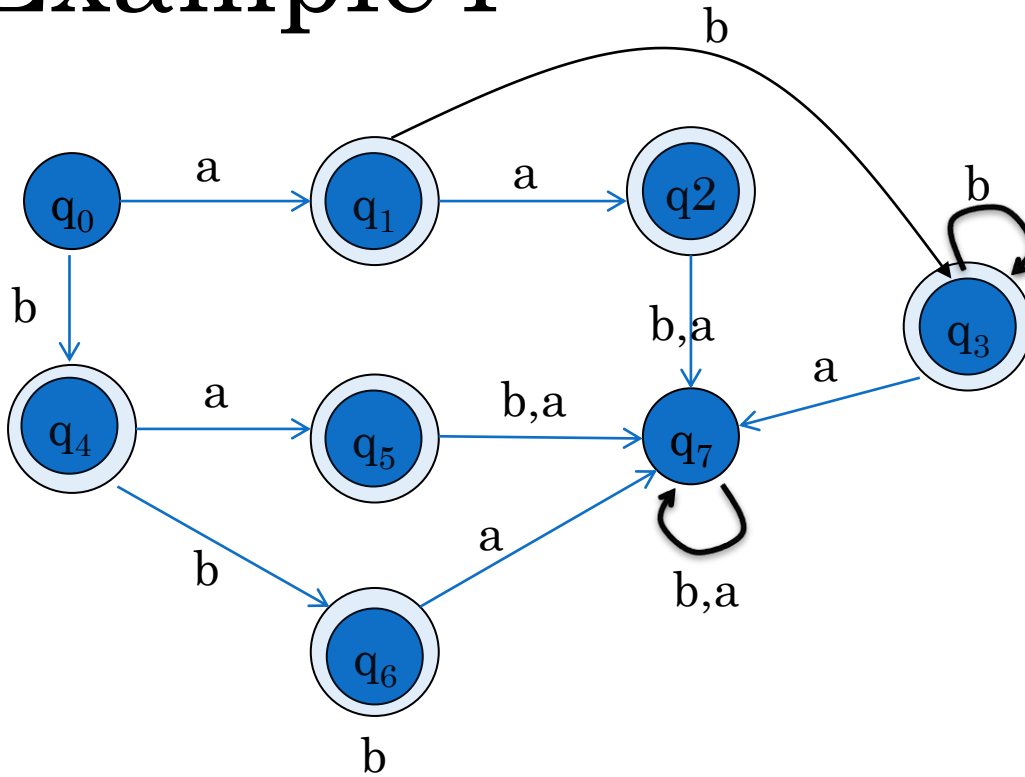
So we have:  $G21 = \{q_0\}$ ,  $G22 = \{q_1, q_2, q_3\}$



# Example 4



# Example4



$G1 = \{q0, q7\}$

$G1 = \{q1, q2, q3, q4, q5, q6\}$

$q0$  is distinguishable,

So we have:  $G21 = \{q0\}$ ,  $G22 = \{q7\}$

$G1$  can be divided to:  $G11 = \{q1, q4\}$  and

$G12 = \{q2, q3, q5, q6\}$

$G12$  can be divided to:  $G121 = \{q3, q6\}$  and

$G122 = \{q2, q5\}$

# Example5

- آیا ادغام پذیر بودن یک رابطه هم ارزی می باشد؟
- ادغام ناپذیر بودن چگونه؟

# Example 6

