Formal languages and automata

Transform NFA to DFA

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NFAs are as powerful as DFAs

- Obviously, an NFA can do everything a DFA can do
- But can it do more?

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- But can it do more?

NO!

- Theorem

A language L is accepted by some DFA if and only if it is accepted by some NFA.

Proof of theorem

- To prove the theorem, we have to show that for every NFA there is a DFA that accepts the same language
- We will give a general method for simulating any NFA by a DFA

NFA to DFA by subset construction

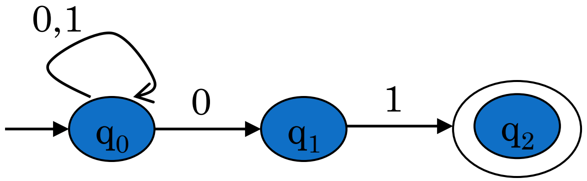
- Let N = $\{Q_N, \sum, \delta_N, q_0, F_N\}$
- Goal: Build D=$\{Q_D, \sum, \delta_D, \{q_0\}, F_D\}$ s.t. L(D)=L(N)
- Construction:

1. Q_D= all subsets of Q_N (i.e., power set)
2. F_D=set of subsets S of Q_N s.t. $S \cap F_N \neq \Phi$
3. δ_D: for each subset S of Q_N and for each input symbol a in $\sum$:
   - $\delta_D(S,a) = \bigcup_{p \text{ in } s} \delta_N(p,a)$

NFA to DFA construction: Example

- $L = \{w \mid w \text{ ends in } 01\}$

NFA:



δ_N	0	1
→ q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
$*q_2$	Ø	Ø

DFA:

NFA to DFA construction: Example

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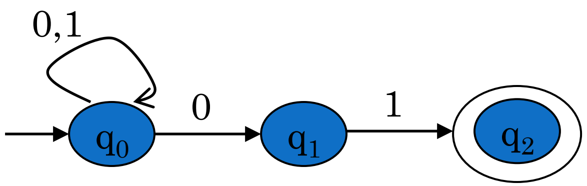
NFA:



δ_N	0	1
→ q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
$*q_2$	Ø	Ø

DFA:

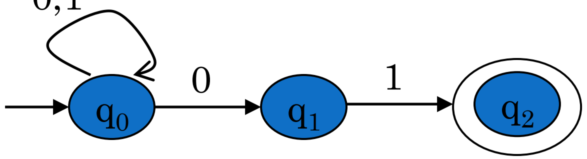
δ_D
Ø
→ $[q_0]$
$[q_1]$
$*[q_2]$
$[q_0,q_1]$
$*[q_0,q_2]$
$*[q_1,q_2]$
$*[q_0,q_1,q_2]$

0. Enumerate all possible subsets

NFA to DFA construction: Example

- *L = {w | w ends in 01}*

NFA:



δ_N	0	1
→ q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
$*q_2$	Ø	Ø

DFA:

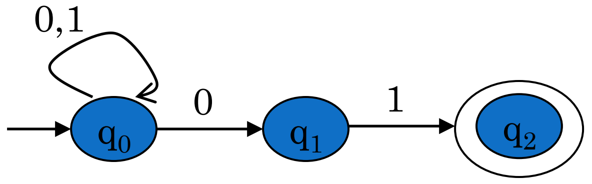
δ_D	0	1
Ø	Ø	Ø
→ $[q_0]$	$\{q_0,q_1\}$	$\{q_0\}$
$[q_1]$	Ø	$\{q_2\}$
$*[q_2]$	Ø	Ø
$[q_0,q_1]$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
$*[q_0,q_2]$	$\{q_0,q_1\}$	$\{q_0\}$
$*[q_1,q_2]$	Ø	$\{q_2\}$
$*[q_0,q_1,q_2]$	$\{q_0,q_1\}$	$\{q_0,q_2\}$

0. Enumerate all possible subsets
1. Determine transitions

NFA to DFA construction: Example

- $L = \{w \mid w \text{ ends in } 01\}$

NFA:



δ_N	0	1
→q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
*q_2	Ø	Ø

DFA:

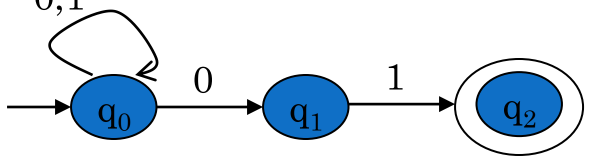
δ_D	0	1
~~Ø~~	~~Ø~~	~~Ø~~
→$[q_0]$	$\{q_0,q_1\}$	$\{q_0\}$
~~$[q_1]$~~	~~Ø~~	~~$\{q_2\}$~~
~~*$[q_2]$~~	~~Ø~~	~~Ø~~
$[q_0,q_1]$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
*$[q_0,q_2]$	$\{q_0,q_1\}$	$\{q_0\}$
~~*$[q_1,q_2]$~~	~~Ø~~	~~$\{q_2\}$~~
~~*$[q_0,q_1,q_2]$~~	~~$\{q_0,q_1\}$~~	~~$\{q_0,q_2\}$~~

0. Enumerate all possible subsets
1. Determine transitions
2. Retain only those states reachable from $\{q_0\}$

NFA to DFA construction: Example

- *L = {w | w ends in 01}*

NFA:



δ_N	0	1
→ q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
*q_2	Ø	Ø

DFA:

δ_D	0	1
~~Ø~~	~~Ø~~	~~Ø~~
→ $[q_0]$	$\{q_0,q_1\}$	$\{q_0\}$
~~$[q_1]$~~	~~Ø~~	~~$\{q_2\}$~~
~~*$[q_2]$~~	~~Ø~~	~~Ø~~
$[q_0,q_1]$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
*$[q_0,q_2]$	$\{q_0,q_1\}$	$\{q_0\}$
~~*$[q_1,q_2]$~~	~~Ø~~	~~$\{q_2\}$~~
~~*$[q_0,q_1,q_2]$~~	~~$\{q_0,q_1\}$~~	~~$\{q_0,q_2\}$~~

δ_D	0	1
→ $[q_0]$	$[q_0,q_1]$	$[q_0]$
$[q_0,q_1]$	$[q_0,q_1]$	$[q_0,q_2]$
*$[q_0,q_2]$	$[q_0,q_1]$	$[q_0]$

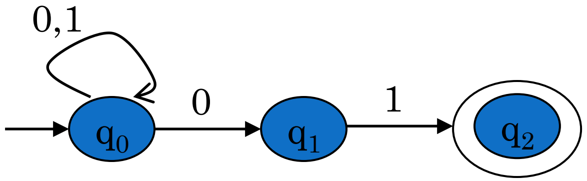
0. Enumerate all possible subsets
1. Determine transitions
2. Retain only those states reachable from $\{q_0\}$

Idea: To avoid enumerating all of power set, do "lazy creation of states"

NFA to DFA construction: Example

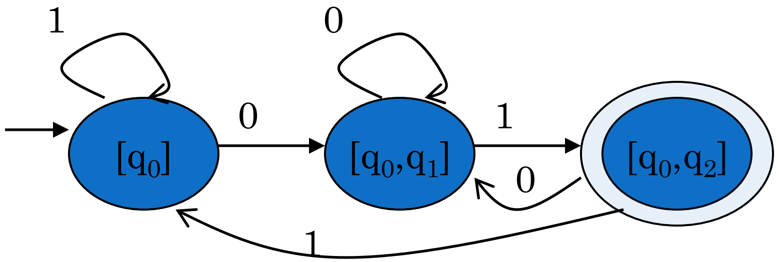
- *L = {w | w ends in 01}*

NFA:



δ_N	0	1
→q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
$*q_2$	Ø	Ø

DFA:



δ_D	0	1
~~Ø~~	~~Ø~~	~~Ø~~
→$[q_0]$	$\{q_0,q_1\}$	$\{q_0\}$
~~$[q_1]$~~	~~Ø~~	~~$\{q_2\}$~~
~~$*[q_2]$~~	~~Ø~~	~~Ø~~
$[q_0,q_1]$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
$*[q_0,q_2]$	$\{q_0,q_1\}$	$\{q_0\}$
~~$*[q_1,q_2]$~~	~~Ø~~	~~$\{q_2\}$~~
~~$*[q_0,q_1,q_2]$~~	~~$\{q_0,q_1\}$~~	~~$\{q_0,q_2\}$~~

⇒

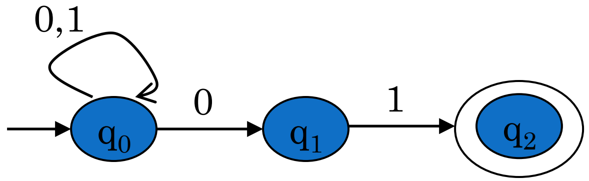
δ_D	0	1
→$[q_0]$	$[q_0,q_1]$	$[q_0]$
$[q_0,q_1]$	$[q_0,q_1]$	$[q_0,q_2]$
$*[q_0,q_2]$	$[q_0,q_1]$	$[q_0]$

0. Enumerate all possible subsets
1. Determine transitions
2. Retain only those states reachable from $\{q_0\}$

NFA to DFA: Repeating the example using *LAZY CREATION*

- *L = {w | w ends in 01}*

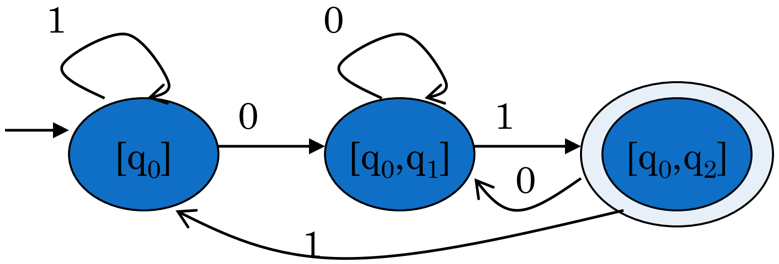
NFA:



δ_N	0	1
→ q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
$*q_2$	Ø	Ø

→

DFA:

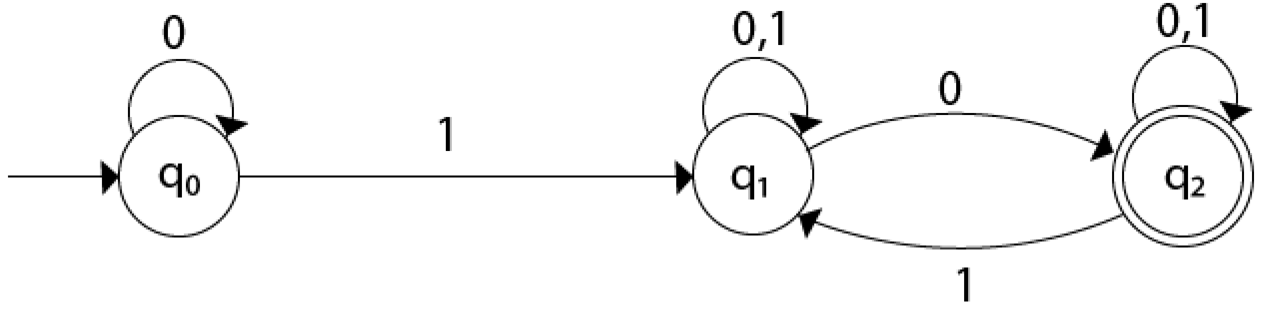


δ_D	0	1
→ $[q_0]$	$[q_0,q_1]$	$[q_0]$

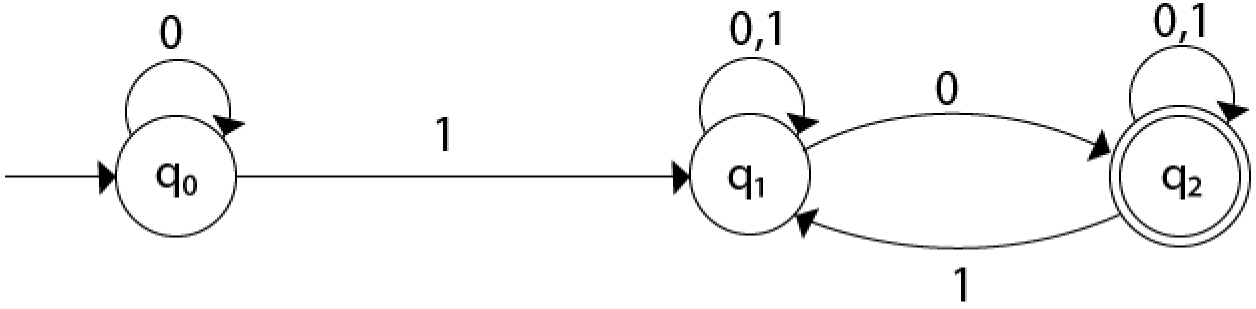
Main Idea:

Introduce states as you go (on a need basis)

Example

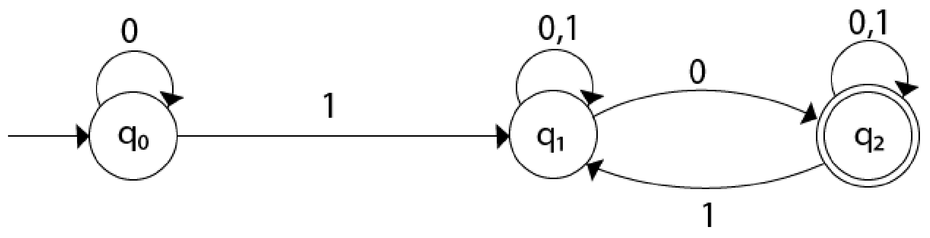


Example



State	0	1
→q0	q0	q1
q1	{q1, q2}	q1

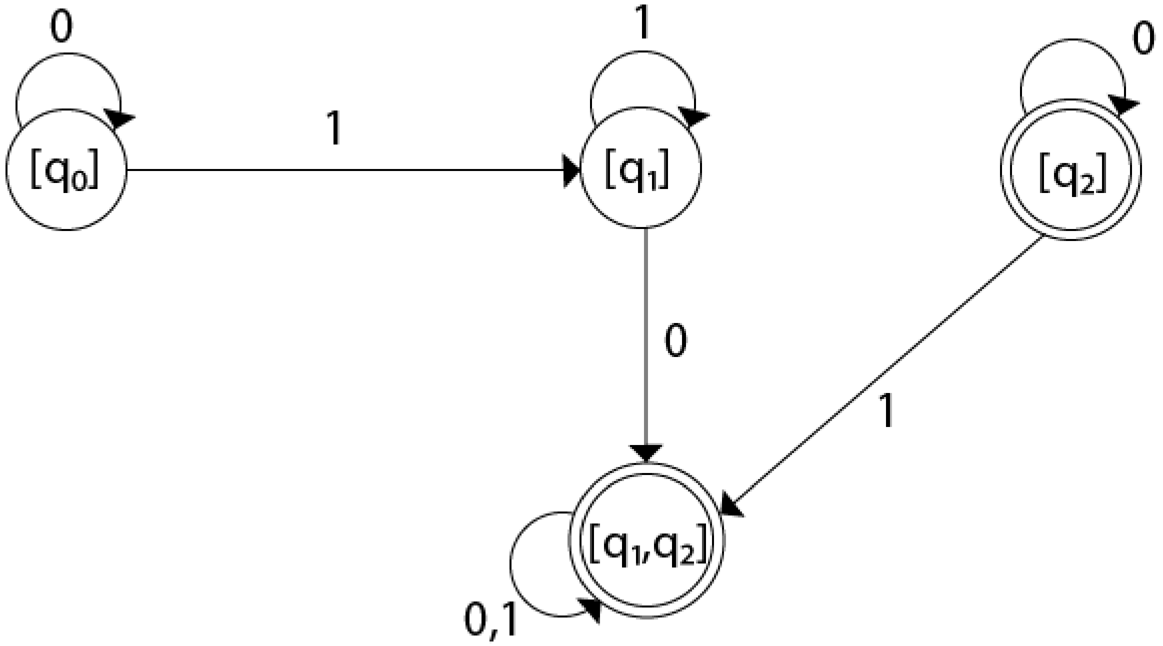
Example



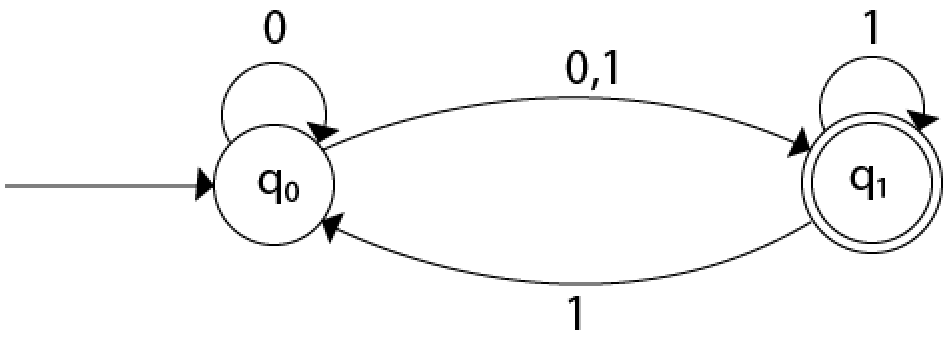
- Now we will obtain δ' transition on $[q1, q2]$

$$
\begin{aligned}
\delta'([q1,q2],0) &= \delta(q1,0) \cup \delta(q2,0) \\
&= \{q1,q2\} \cup \{q2\} \\
&= [q1,q2] \\
\delta'([q1,q2],1) &= \delta(q1,1) \cup \delta(q2,1) \\
&= \{q1\} \cup \{q1,q2\} \\
&= \{q1,q2\} \\
&= [q1,q2]
\end{aligned}
$$

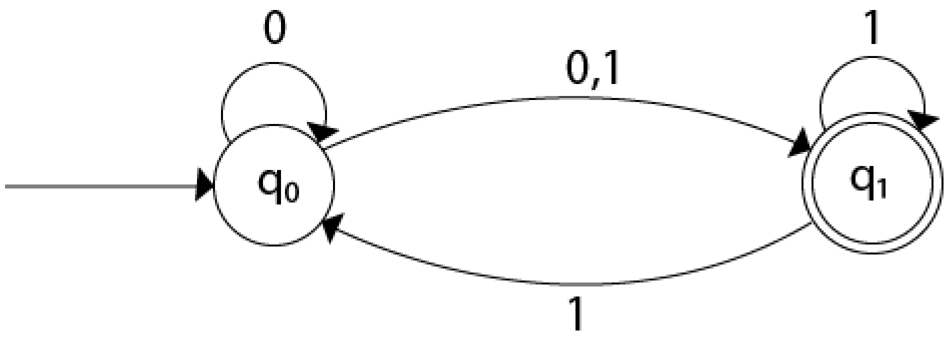
Example



Example

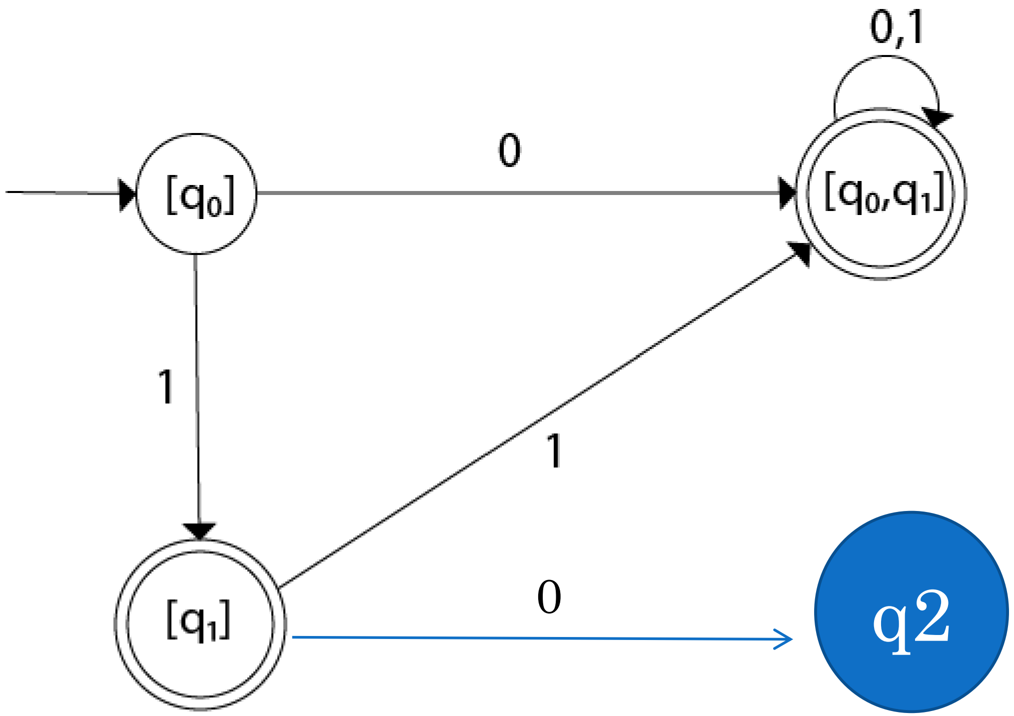


Example



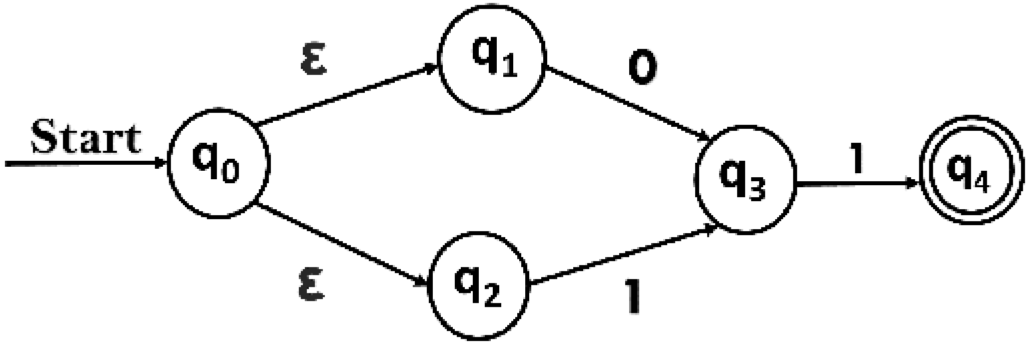
State	0	1
→[q0]	[q0, q1]	[q1]
*[q1]	φ	[q0, q1]
*[q0, q1]	[q0, q1]	[q0, q1]

Example



Conversion from NFA with ε to DFA

- **Definition:**
- **ε-closure:** ε-closure for a given state A means a set of states which can be reached from the state A with only ε(null) move including the state A itself.



Conversion from NFA with ε to DFA

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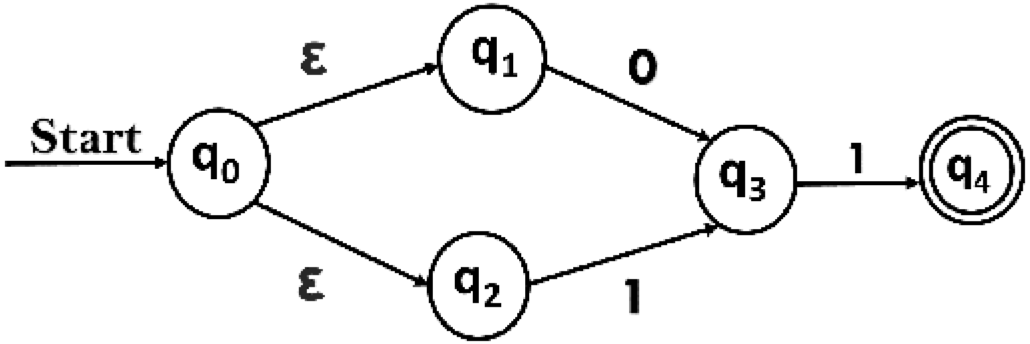
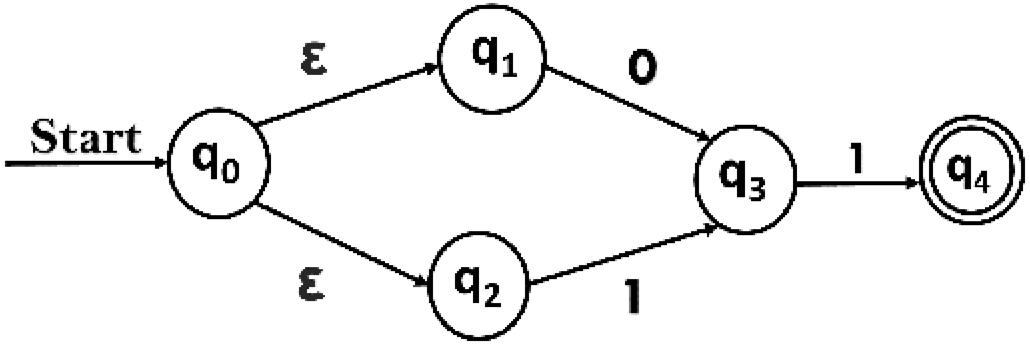


ε-closure {q0} = {q0, q1, q2}
ε-closure {q1} = {q1}
ε-closure {q2} = {q2}
ε-closure {q3} = {q3}
ε-closure {q4} = {q4}

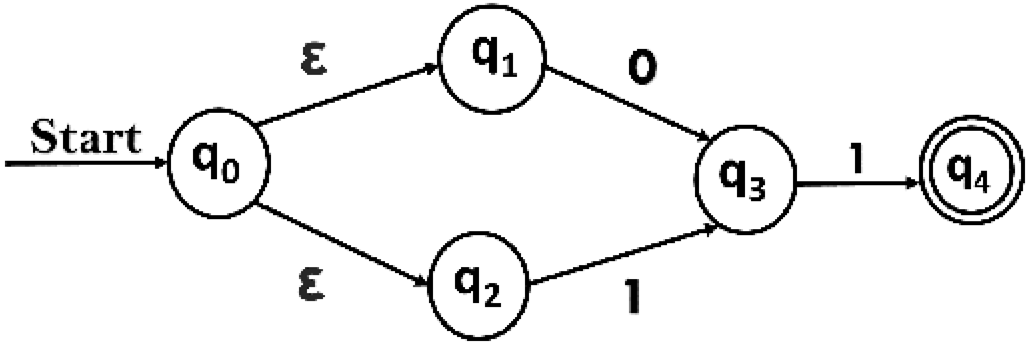
Conversion from NFA with ε to DFA

- **Steps for converting NFA with ε to DFA:**
- **Step 1:** We will take the ε-closure for the starting state of NFA as a starting state of DFA.
- **Step 2:** Find the states for each input symbol that can be traversed from the present. That means the union of transition value and their closures for each state of NFA present in the current state of DFA.
- **Step 3:** If we found a new state, take it as current state and repeat step 2.
- **Step 4:** Repeat Step 2 and Step 3 until there is no new state present in the transition table of DFA.
- **Step 5:** Mark the states of DFA as a final state which contains the final state of NFA.

Example

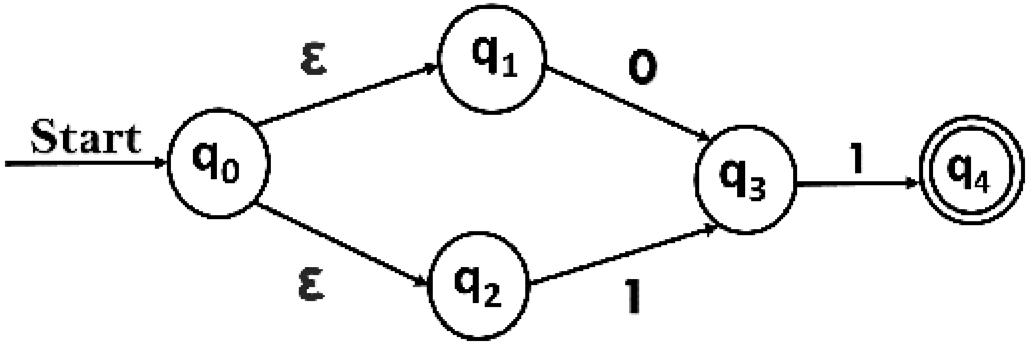


Example

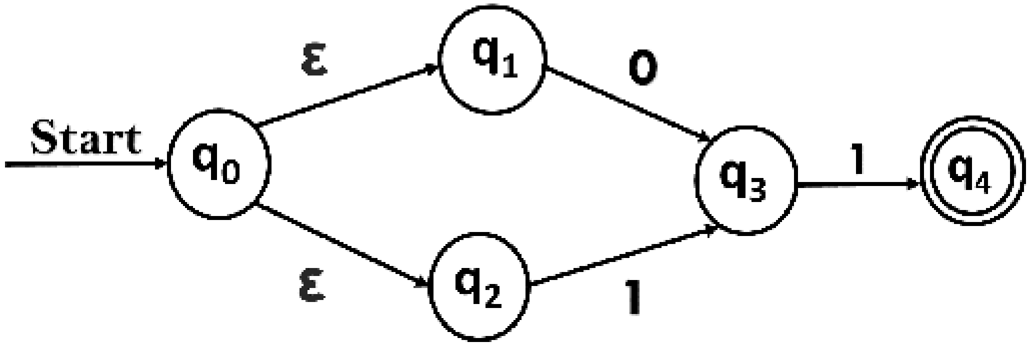


Now, let ε-closure {q0} = {q0, q1, q2} be state A.

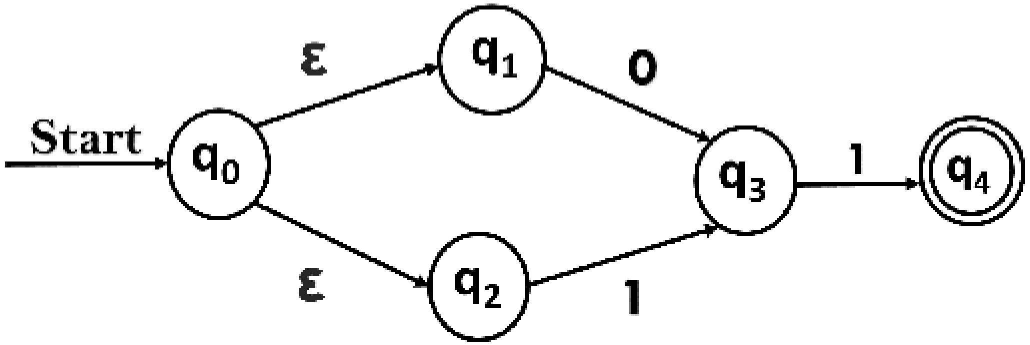
Example




```
δ'(A, 0) = ε-closure {δ((q0, q1, q2), 0) }
              = ε-closure {δ(q0, 0) ∪ δ(q1, 0) ∪ δ(q2, 0) }
              = ε-closure {q3}
              = {q3}            call it as state B.

δ'(A, 1) = ε-closure {δ((q0, q1, q2), 1) }
              = ε-closure {δ((q0, 1) ∪ δ(q1, 1) ∪ δ(q2, 1) }
              = ε-closure {q3}
              = {q3} = B.
```




```
δ'(B, 0) = ε-closure {δ(q3, 0) }
                = φ
δ'(B, 1) = ε-closure {δ(q3, 1) }
                = ε-closure {q4}
                = {q4}              i.e. state C
```



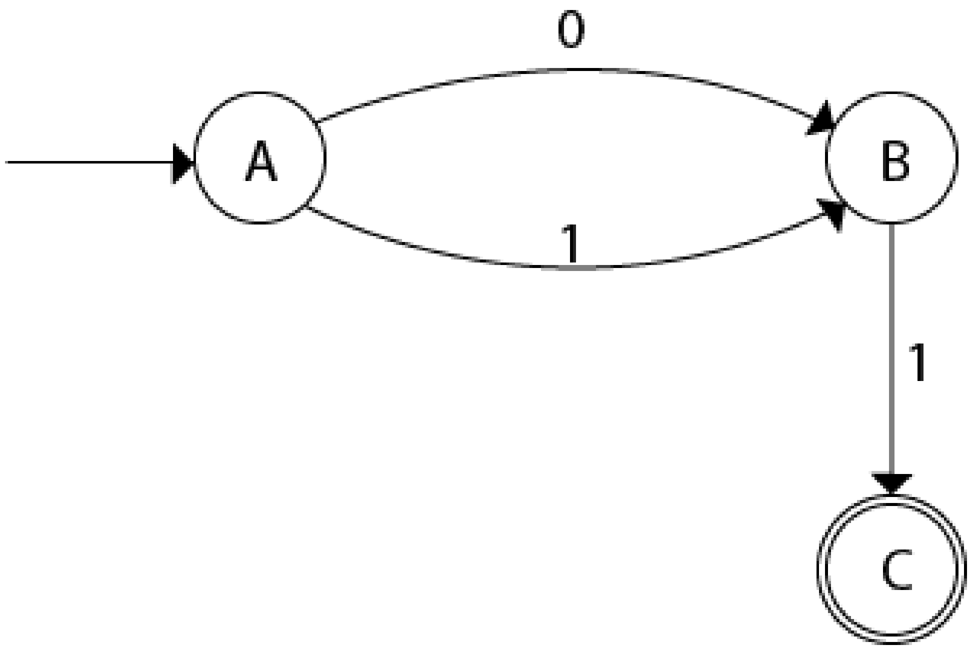
$\delta'(C, 0) = \varepsilon\text{-closure } \{\delta(q4, 0)\}$

$= \phi$

$\delta'(C, 1) = \varepsilon\text{-closure } \{\delta(q4, 1)\}$

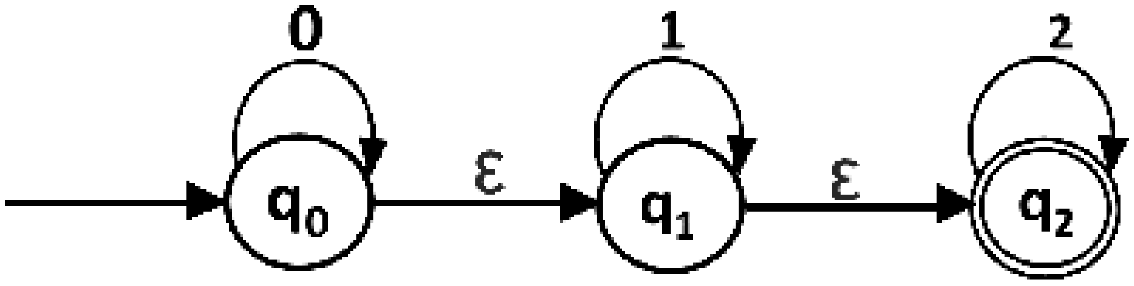
$= \phi$

- The DFA will be,



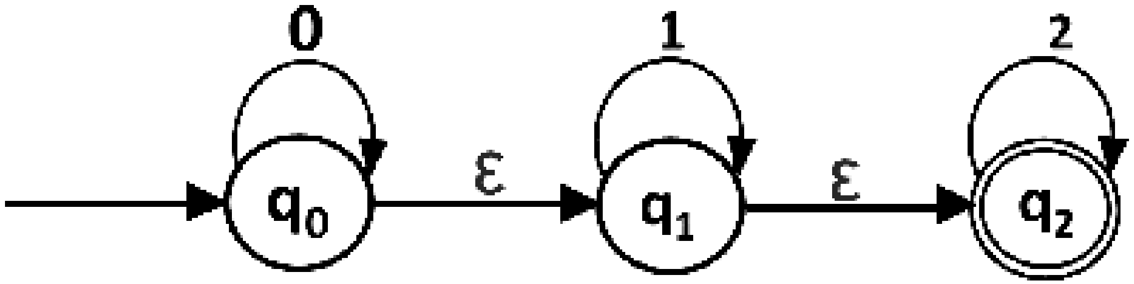
Example

- Convert the given NFA into its equivalent DFA.



Example

- Convert the given NFA into its equivalent DFA.



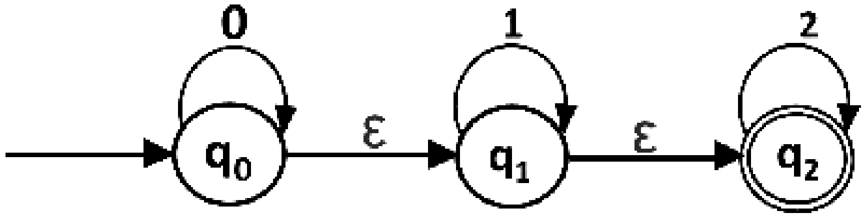
Let us obtain the ε-closure of each state.

ε-closure(q0) = {q0, q1, q2}

ε-closure(q1) = {q1, q2}

ε-closure(q2) = {q2}

Example



- Now we will obtain δ' transition. Let ε-closure(q0) = {q0, q1, q2} call it as **state A**.

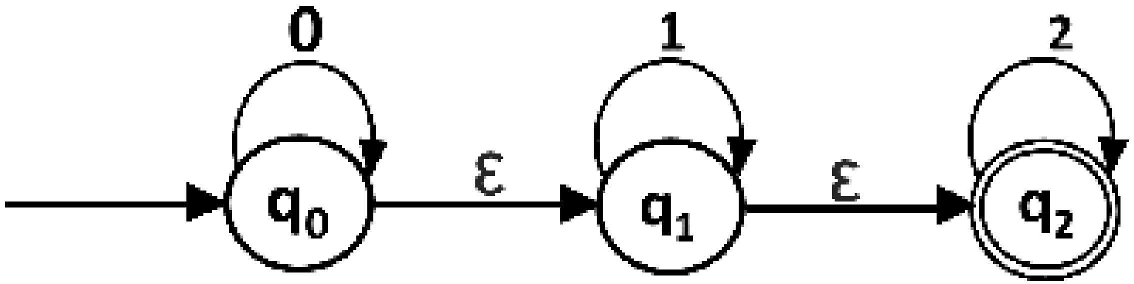
```
δ'(A, 0) = ε-closure{δ((q0, q1, q2), 0)}
              = ε-closure{δ(q0, 0) ∪ δ(q1, 0) ∪ δ(q2, 0)}
              = ε-closure{q0}
              = {q0, q1, q2}
```

```
δ'(A, 1) = ε-closure{δ((q0, q1, q2), 1)}
              = ε-closure{δ(q0, 1) ∪ δ(q1, 1) ∪ δ(q2, 1)}
              = ε-closure{q1}
              = {q1, q2}         call it as state B
```

Example

```
δ'(A, 2) = ε-closure{δ((q0, q1, q2), 2)}
                = ε-closure{δ(q0, 2) ∪ δ(q1, 2) ∪ δ(q2, 2)}
                = ε-closure{q2}
                = {q2}          call it state C
```

Example

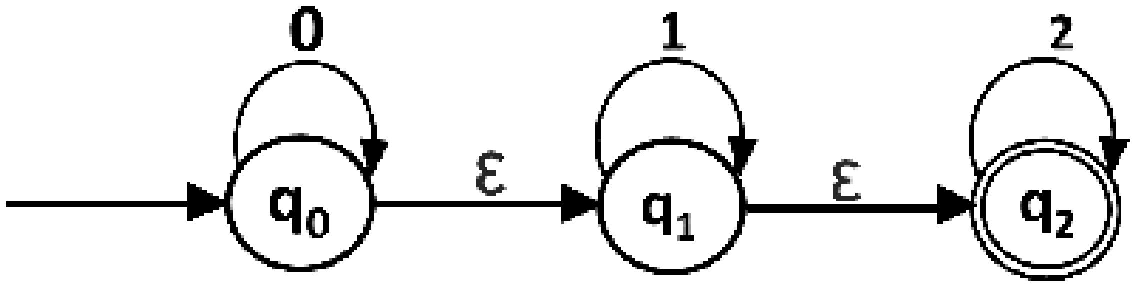


- Now we will find the transitions on states B and C for each input.

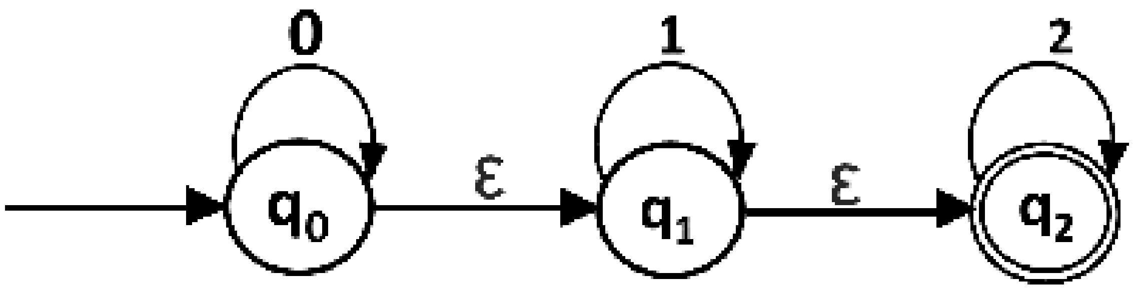
```
δ'(B, 0) = ε-closure{δ((q1, q2), 0)}
              = ε-closure{δ(q1, 0) ∪ δ(q2, 0)}
              = ε-closure{φ}
              = φ
```

```
δ'(B, 1) = ε-closure{δ((q1, q2), 1)}
              = ε-closure{δ(q1, 1) ∪ δ(q2, 1)}
              = ε-closure{q1}
              = {q1, q2}         i.e. state B itself
```

Example



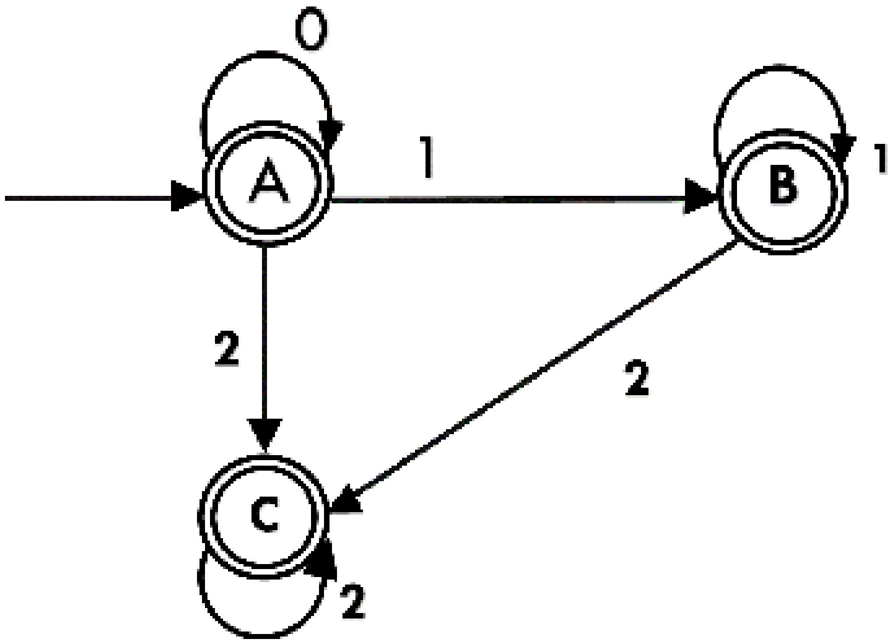

```
δ'(B, 2) = ε-closure{δ((q1, q2), 2)}
                   = ε-closure{δ(q1, 2) ∪ δ(q2, 2)}
                   = ε-closure{q2}
                   = {q2}          i.e. state C itself
```



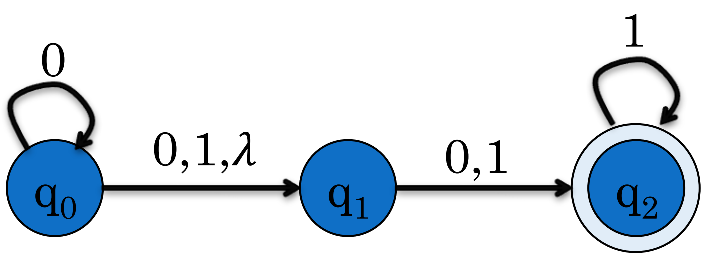
- Now we will obtain transitions for C:

```
δ'(C, 0) = ε-closure{δ(q2, 0)}
               = ε-closure{φ}
               = φ

δ'(C, 1) = ε-closure{δ(q2, 1)}
               = ε-closure{φ}
               = φ

δ'(C, 2) = ε-closure{δ(q2, 2)}
               = {q2}
```

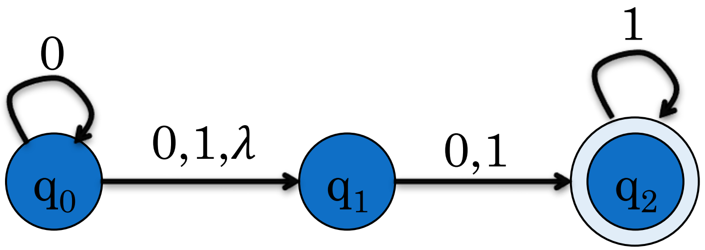
- Hence we have:



Homework



Solution



$\lambda - closure(q_0) = \{q_0, q_1\}$
$\lambda - closure(q_1) = \{q_1\}$
$\lambda - closure(q_2) = \{q_2\}$

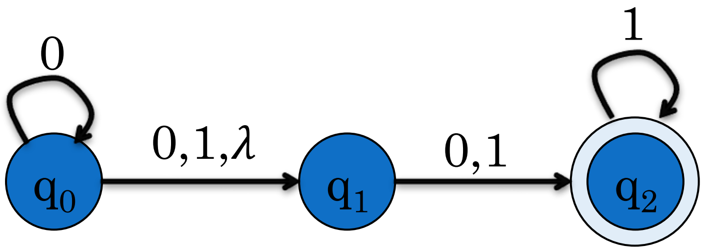
$\boldsymbol{\delta}(\{\boldsymbol{q_0}, \boldsymbol{q_1}\}, \mathbf{0}) = \lambda - closure(\delta(\{q_0, q_1\}, 0)) = \lambda - closure(\delta(\{q_0\}, 0) \cup \delta(\{q_1\}, 0)) =$
$\lambda - closue(\{q_0, q_1\} \cup \{q_2\}) = \{q_0, q_1, q_2\}$

$\boldsymbol{\delta}(\{\boldsymbol{q_0}, \boldsymbol{q_1}\}, \mathbf{1}) = \lambda - closure(\delta(\{q_0, q_1\}, 1)) = \lambda - closure(\delta(\{q_0\}, 1) \cup \delta(\{q_1\}, 1)) =$
$\lambda - closue(\{q_1\} \cup \{q_2\}) = \{q_2, q_1\}$

$\boldsymbol{\delta}(\{\boldsymbol{q_2}, \boldsymbol{q_1}\}, \mathbf{0}) = \lambda - closure(\delta(\{q_2, q_1\}, 0)) = \lambda - closure(\delta(\{q_2\}, 0) \cup$
$\delta(\{q_1\}, 0)) = \lambda - closue(\{\quad\} \cup \{q_2\}) = \{q_2\}$

$\boldsymbol{\delta}(\{\boldsymbol{q_2}, \boldsymbol{q_1}\}, \mathbf{1}) = \lambda - closure(\delta(\{q_2, q_1\}, 1)) = \lambda - closure(\delta(\{q_2\}, 1) \cup$
$\delta(\{q_1\}, 1)) = \lambda - closue(\{q_2\} \cup \{q_2\}) = \{q_2\}$

Solution



$\lambda - closure(q_0) = \{q_0, q_1\}$
$\lambda - closure(q_1) = \{q_1\}$
$\lambda - closure(q_2) = \{q_2\}$

$\delta(\{q_0, q_1, q_2\}, 0) = \lambda - closure(\delta(\{q_0, q_1, q_2, \}, 0)) = \{q_0, q_1, q_2\}$

$\boldsymbol{\delta}(\{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_2}\}, \mathbf{1}) = \{q_2, q_1\}$

$\boldsymbol{\delta}(\{\boldsymbol{q_2}\}, \mathbf{0}) = \lambda - closure(\delta(\{q_2\}, 0)) = \{\}$

$\boldsymbol{\delta}(\{\boldsymbol{q_2}\}, \mathbf{1}) = \lambda - closure(\delta(\{q_2\}, 1)) = \{q_2\}$